Postulates, Theorems, and Constructions

Chapter 1: Tools of Geometry

Postulate 1-1
Through any two points there is exactly one line. (p. 18)

Postulate 1-2
If two lines intersect, then they intersect in exactly one point. (p. 18)

Postulate 1-3
If two planes intersect, then they intersect in exactly one line. (p. 18)

Postulate 1-4
Through any three noncollinear points there is exactly one plane. (p. 19)

Postulate 1-5
Ruler Postulate
The points of a line can be put into one-to-one correspondence with the real numbers so that the distance between any two points is the absolute value of the difference of the corresponding numbers. (p. 31)

Postulate 1-6
Segment Addition Postulate
If three points A, B, and C are collinear and B is between A and C, then $AB + BC = AC$. (p. 32)

Postulate 1-7
Protractor Postulate
Let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be opposite rays in a plane. $\overrightarrow{OA}$, $\overrightarrow{OB}$, and all the rays with endpoint O that can be drawn on one side of $\overleftrightarrow{AB}$ can be paired with the real numbers from 0 to 180 so that

a. $\overrightarrow{OA}$ is paired with 0 and $\overrightarrow{OB}$ is paired with 180.
b. If $\overrightarrow{OC}$ is paired with x and $\overrightarrow{OD}$ is paired with y, then $m\angle COD = |x - y|$. (p. 37)

Postulate 1-8
Angle Addition Postulate
If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.
If $\angle AOC$ is a straight angle, then $m\angle AOB + m\angle BOC = 180$. (p. 38)

The Distance Formula
The distance d between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. (p. 53)
- Proof on p. 421, Exercise 34

The Midpoint Formula
The coordinates of the midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ are the following.
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (p. 55)

The Distance Formula (Three Dimensions)
In a three-dimensional coordinate system, the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) can be found using this extension of the Distance Formula.
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ (p. 58)

Postulate 1-9
If two figures are congruent, then their areas are equal. (p. 64)

Postulate 1-10
The area of a region is the sum of the areas of its nonoverlapping parts. (p. 64)

Chapter 2: Reasoning and Proof

Law of Detachment
If a conditional is true and its hypothesis is true, then its conclusion is true. In symbolic form: If $p \rightarrow q$ is a true statement and p is true, then q is true. (p. 95)

Law of Syllogism
If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement. (p. 95)

Properties of Congruence
Reflexive Property
$\overline{AB} \cong \overline{AB}$ and $\angle A \cong \angle A$
Symmetric Property
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
Transitive Property
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$. (p. 105)

Theorem 2-1
Vertical Angles Theorem
Vertical angles are congruent. (p. 110)
- Proof on p. 111

Theorem 2-2
Congruent Supplements Theorem
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent. (p. 111)
- Proofs on p. 112, Example 2; p. 114, Exercise 27

Theorem 2-3
Congruent Complements Theorem
If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent. (p. 112)
- Proofs on p. 113, Exercise 7; p. 114, Exercise 28

Theorem 2-4
All right angles are congruent. (p. 112)
- Proof on p. 113, Exercise 14

Theorem 2-5
If two angles are congruent and supplementary, then each is a right angle. (p. 112)
- Proof on p. 114, Exercise 21

Chapter 3: Parallel and Perpendicular Lines

Postulate 3-1
Corresponding Angles Postulate
If a transversal intersects two parallel lines, then corresponding angles are congruent. (p. 128)

Theorem 3-1
Alternate Interior Angles Theorem
If a transversal intersects two parallel lines, then alternate interior angles are congruent. (p. 128)
- Proof on p. 129

Theorem 3-2
Same-Side Interior Angles Theorem
If a transversal intersects two parallel lines, then same-side interior angles are supplementary. (p. 128)
- Proof on p. 132, Exercise 29

Theorem 3-3
Alternate Exterior Angles Theorem
If a transversal intersects two parallel lines, then alternate exterior angles are congruent. (p. 130)
- Proof on p. 129, Example 3

Theorem 3-4
Same-Side Exterior Angles Theorem
If a transversal intersects two parallel lines, then same-side exterior angles are supplementary. (p. 130)
- Proof on p. 129, Quick Check 3

Postulate 3-2
Converse of the Corresponding Angles Postulate
If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel. (p. 134)

Theorem 3-5
Converse of the Alternate Interior Angles Theorem
If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel. (p. 135)
- Proof on p. 135

Theorem 3-6
Converse of the Same-Side Interior Angles Theorem
If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel. (p. 135)
- Proofs on p. 138, Exercise 22 and p. 139, Exercise 40

Theorem 3-7
Converse of the Alternate Exterior Angles Theorem
If two lines and a transversal form alternate exterior angles that are congruent, then the lines are parallel. (p. 136)
- Proof on p. 136

Theorem 3-8
Converse of the Same-Side Exterior Angles Theorem
If two lines and a transversal form same-side exterior angles that are supplementary, then the lines are parallel. (p. 136)
- Proof on p. 138, Exercise 27

Theorem 3-9
If two lines are parallel to the same line, then they are parallel to each other. (p. 141)
- Proofs on p. 143, Exercise 3; p. 179, Exercise 37

Theorem 3-10
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other. (p. 141)
- Proofs on p. 141; p. 143, Exercise 12; p. 179, Exercise 38

Theorem 3-11
In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other. (p. 142)
- Proof on p. 143, Exercise 11

Theorem 3-12
Triangle Angle-Sum Theorem
The sum of the measures of the angles of a triangle is 180. (p. 147)
- Proof on p. 147

Theorem 3-13
Triangle Exterior Angle Theorem
The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles. (p. 149)
- Proof on p. 152, Exercise 35

  Corollary
  The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles. (p. 290)
  - Proof on p. 290

Parallel Postulate
Through a point not on a line, there is one and only one line parallel to the given line. (p. 154)

Spherical Geometry Parallel Postulate
Through a point not on a line, there is no line parallel to the given line. (p. 154)

Theorem 3-14
Polygon Angle-Sum Theorem
The sum of the measures of the angles of an n-gon is $(n - 2)180$. (p. 159)
- Proof on p. 163, Exercise 54

Theorem 3-15
Polygon Exterior Angle-Sum Theorem
The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360. (p. 160)
- Proofs on p. 156 (using a computer) and p. 162, Exercise 46

Slopes of Parallel Lines
If two nonvertical lines are parallel, their slopes are equal. If the slopes of two distinct nonvertical lines are equal, the lines are parallel. Any two vertical lines are parallel. (p. 174)
- Proofs on pp. 387–388, Exercises 42, 43

Slopes of Perpendicular Lines
If two nonvertical lines are perpendicular, the product of their slopes is -1. If the slopes of two lines have a product of -1, the lines are perpendicular. Any horizontal line and vertical line are perpendicular. (p. 175)
- Proofs on p. 346, Exercise 29 and p. 353, Exercise 41

Chapter 4: Congruent Triangles

Theorem 4-1
If the two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent. (p. 199)
- Proof on p. 202, Exercise 45

Postulate 4-1
Side-Side-Side (SSS) Postulate
If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent. (p. 205)

Postulate 4-2
Side-Angle-Side (SAS) Postulate
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent. (p. 206)

Postulate 4-3
Angle-Side-Angle (ASA) Postulate
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent. (p. 213)

Theorem 4-2
Angle-Angle-Side (AAS) Theorem
If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent. (p. 214)
- Proof on p. 214

Theorem 4-3
Isosceles Triangle Theorem
If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (p. 228)
- Proofs on p. 229; p. 231, Exercise 15

  Corollary
  If a triangle is equilateral, then the triangle is equiangular. (p. 230)
  - Proof on p. 231, Exercise 18

Theorem 4-4
Converse of the Isosceles Triangle Theorem
If two angles of a triangle are congruent, then the sides opposite the angles are congruent. (p. 228)
- Proof on p. 231, Exercise 16

  Corollary
  If a triangle is equiangular, then the triangle is equilateral. (p. 230)
  - Proof on p. 231, Exercise 18

Theorem 4-5
The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base. (p. 228)
- Proof on p. 232, Exercise 29

Theorem 4-6
Hypotenuse-Leg (HL) Theorem
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. (p. 235)
- Proof on p. 235

Chapter 5: Relationships Within Triangles

Theorem 5-1
Triangle Midsegment Theorem
If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side, and is half its length. (p. 260)
- Proof on p. 260

Theorem 5-2
Perpendicular Bisector Theorem
If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. (p. 265)
- Proof on p. 269, Exercise 40

Theorem 5-3
Converse of the Perpendicular Bisector Theorem
If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. (p. 265)
- Proof on p. 269, Exercise 41

Theorem 5-4
Angle Bisector Theorem
If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle. (p. 266)
- Proof on p. 269, Exercise 43

Theorem 5-5
Converse of the Angle Bisector Theorem
If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector. (p. 266)
- Proof on p. 269, Exercise 44

Theorem 5-6
The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices. (p. 273)
- Proof on p. 277, Exercise 29

Theorem 5-7
The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides. (p. 273)
- Proof on p. 277, Exercise 30

Theorem 5-8
The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side. (p. 274)
- Proof on p. 352, Exercise 35

Theorem 5-9
The lines that contain the altitudes of a triangle are concurrent. (p. 275)
- Proof on p. 352, Exercise 36

Comparison Property of Inequality
If $a = b + c$ and $c > 0$, then $a > b$. (p. 289)
- Proof on p. 289

Theorem 5-10
If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side. (p. 290)
- Proof on p. 294, Exercise 33

Theorem 5-11
If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle. (p. 291)
- Proof on p. 291

Theorem 5-12
Triangle Inequality Theorem
The sum of the lengths of any two sides of a triangle is greater than the length of the third side. (p. 292)
- Proof on p. 294, Exercise 41

Chapter 6: Quadrilaterals

Theorem 6-1
Opposite sides of a parallelogram are congruent. (p. 312)
- Proofs on p. 312; p. 317, Exercise 35

Theorem 6-2
Opposite angles of a parallelogram are congruent. (p. 313)
- Proof on p. 317, Exercise 36

Theorem 6-3
The diagonals of a parallelogram bisect each other. (p. 314)
- Proofs on p. 314; p. 350, Exercise 2

Theorem 6-4
If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal. (p. 315)
- Proof on p. 318, Exercise 52

Theorem 6-5
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 321)
- Proof on p. 321

Theorem 6-6
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 321)
- Proof on p. 325, Exercise 12

Theorem 6-7
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (p. 322)
- Proof on p. 322

Theorem 6-8
If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram. (p. 322)
- Proof on p. 325, Exercise 18

Theorem 6-9
Each diagonal of a rhombus bisects two angles of the rhombus. (p. 329)
- Proof on p. 329

Theorem 6-10
The diagonals of a rhombus are perpendicular. (p. 330)
- Proof on p. 333, Exercise 22

Theorem 6-11
The diagonals of a rectangle are congruent. (p. 330)
- Proof on p. 330

Theorem 6-12
If one diagonal of a parallelogram bisects two angles of the parallelogram, then the parallelogram is a rhombus. (p. 331)
- Proof on p. 334, Exercise 39

Theorem 6-13
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (p. 331)
- Proof on p. 333, Exercise 23

Theorem 6-14
If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (p. 331)
- Proof on p. 334, Exercise 40

Theorem 6-15
The base angles of an isosceles trapezoid are congruent. (p. 336)
- Proof on p. 340, Exercise 38

Theorem 6-16
The diagonals of an isosceles trapezoid are congruent. (p. 337)
- Proofs on p. 337; p. 350, Exercise 3

Theorem 6-17
The diagonals of a kite are perpendicular. (p. 338)
- Proof on p. 338

Theorem 6-18
Trapezoid Midsegment Theorem
(1) The midsegment of a trapezoid is parallel to the bases.
(2) The length of a midsegment of a trapezoid is half the sum of the lengths of the bases. (p. 348)
- Proof on p. 349, Quick Check 1

Chapter 7: Similarity

Postulate 7-1
Angle-Angle Similarity (AA ~) Postulate
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. (p. 382)

Theorem 7-1
Side-Angle-Side Similarity (SAS ~) Theorem
If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then the triangles are similar. (p. 383)
- Proof on p. 383

Theorem 7-2
Side-Side-Side Similarity (SSS ~) Theorem
If the corresponding sides of two triangles are proportional, then the triangles are similar. (p. 383)
- Proof on p. 383

Theorem 7-3
The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other. (p. 392)
- Proof on p. 392

Corollary 1
The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse. (p. 392)
- Proof on p. 392

Corollary 2
The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the adjacent hypotenuse segment and the length of the hypotenuse. (p. 393)
- Proof on p. 393

Theorem 7-4
Side-Splitter Theorem
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally. (p. 398)
- Proof on p. 398

Converse
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.
- Proof on p. 402, Exercise 34

Corollary
If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional. (p. 399)
- Proof on p. 402, Exercise 35

Theorem 7-5
Triangle-Angle-Bisector Theorem
If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle. (p. 400)
- Proof on p. 400

Chapter 8: Right Triangles and Trigonometry

Theorem 8-1
Pythagorean Theorem
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.
$a^2 + b^2 = c^2$ (p. 417)
- Proofs on p. 395, Exercise 53; p. 416; p. 422, Exercise 48; p. 545; p. 692, Exercise 36

Theorem 8-2
Converse of the Pythagorean Theorem
If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle. (p. 419)
- Proof on p. 423, Exercise 58

Theorem 8-3
If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, the triangle is obtuse. (p. 419)

Theorem 8-4
If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, the triangle is acute. (p. 419)

Theorem 8-5
45°-45°-90° Triangle Theorem
In a 45°-45°-90° triangle, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.
hypotenuse $= \sqrt{2} \cdot$ leg (p. 425)
- Proof on p. 425

Theorem 8-6
30°-60°-90° Triangle Theorem
In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.
hypotenuse $= 2 \cdot$ shorter leg
longer leg $= \sqrt{3} \cdot$ shorter leg (p. 426)
- Proof on p. 427

Chapter 9: Transformations

Theorem 9-1
A translation or rotation is a composition of two reflections. (p. 506)

Theorem 9-2
A composition of reflections across two parallel lines is a translation. (p. 507)

Theorem 9-3
A composition of reflections across two intersecting lines is a rotation. (p. 507)

Theorem 9-4
Fundamental Theorem of Isometries
In a plane, one of two congruent figures can be mapped onto the other by a composition of at most three reflections. (p. 508)

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Theorem 9-5
Isometry Classification Theorem
There are only four isometries. They are reflection, translation, rotation, and glide reflection. (p. 509)

Theorem 9-6
Every triangle tessellates. (p. 516)

Theorem 9-7
Every quadrilateral tessellates. (p. 516)

Chapter 10: Area

Theorem 10-1
Area of a Rectangle
The area of a rectangle is the product of its base and height.
$A = bh$ (p. 534)

Theorem 10-2
Area of a Parallelogram
The area of a parallelogram is the product of a base and the corresponding height.
$A = bh$ (p. 534)

Theorem 10-3
Area of a Triangle
The area of a triangle is half the product of a base and the corresponding height.
$A = \frac{1}{2}bh$ (p. 535)

Theorem 10-4
Area of a Trapezoid
The area of a trapezoid is half the product of the height and the sum of the bases.
$A = \frac{1}{2}h(b_1 + b_2)$ (p. 540)

Theorem 10-5
Area of a Rhombus or a Kite
The area of a rhombus or a kite is half the product of the lengths of its diagonals.
$A = \frac{1}{2}d_1d_2$ (p. 541)

Theorem 10-6
Area of a Regular Polygon
The area of a regular polygon is half the product of the apothem and the perimeter.
$A = \frac{1}{2}ap$ (p. 547)

Theorem 10-7
Perimeters and Areas of Similar Figures
If the similarity ratio of two similar figures is $\frac{a}{b}$, then
(1) the ratio of their perimeters is $\frac{a}{b}$ and
(2) the ratio of their areas is $\frac{a^2}{b^2}$. (p. 554)

Theorem 10-8
Area of a Triangle Given SAS
The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.
Area of $\triangle ABC = \frac{1}{2}bc(\sin A)$ (p. 561)
- Proof on p. 560

Postulate 10-1
Arc Addition Postulate
The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs. (p. 567)

Theorem 10-9
Circumference of a Circle
The circumference of a circle is π times the diameter.
$C = \pi d$ or $C = 2\pi r$ (p. 568)

Theorem 10-10
Arc Length
The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle.
length of $\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$ (p. 569)

Theorem 10-11
Area of a Circle
The area of a circle is the product of π and the square of the radius. $A = \pi r^2$ (p. 576)

Theorem 10-12
Area of a Sector of a Circle
The area of a sector of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the area of the circle.
Area of sector $AOB = \frac{m\widehat{AB}}{360} \cdot \pi r^2$ (p. 576)

Chapter 11: Surface Area and Volume

Theorem 11-1
Lateral and Surface Areas of a Prism
The lateral area of a right prism is the product of the perimeter of the base and the height.
L.A. $= ph$
The surface area of a right prism is the sum of the lateral area and the areas of the two bases.
S.A. $=$ L.A. $+ 2B$ (p. 610)

Theorem 11-2
Lateral and Surface Areas of a Cylinder
The lateral area of a right cylinder is the product of the circumference of the base and the height of the cylinder.
L.A. $= 2\pi rh$, or L.A. $= \pi dh$
The surface area of a right cylinder is the sum of the lateral area and the areas of the two bases.
S.A. $=$ L.A. $+ 2B$, or S.A. $= 2\pi rh + 2\pi r^2$ (p. 610)

Theorem 11-3
Lateral and Surface Areas of a Regular Pyramid
The lateral area of a regular pyramid is half the product of the perimeter of the base and the slant height.
L.A. $= \frac{1}{2}p\ell$
The surface area of a regular pyramid is the sum of the lateral area and the area of the base.
S.A. $=$ L.A. $+ B$ (p. 618)

Theorem 11-4
Lateral and Surface Areas of a Cone
The lateral area of a right cone is half the product of the circumference of the base and the slant height.
L.A. $= \frac{1}{2} \cdot 2\pi r\ell$, or L.A. $= \pi r\ell$
The surface area of a right cone is the sum of the lateral area and the area of the base.
S.A. $=$ L.A. $+ B$ (p. 619)

Theorem 11-5
Cavalieri's Principle
If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume. (p. 625)

Theorem 11-6
Volume of a Prism
The volume of a prism is the product of the area of a base and the height of the prism.
$V = Bh$ (p. 625)

Theorem 11-7
Volume of a Cylinder
The volume of a cylinder is the product of the area of the base and the height of the cylinder.
$V = Bh$, or $V = \pi r^2h$ (p. 626)

Theorem 11-8
Volume of a Pyramid
The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.
$V = \frac{1}{3}Bh$ (p. 632)

Theorem 11-9
Volume of a Cone
The volume of a cone is one third the product of the area of the base and the height of the cone.
$V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2h$ (p. 633)

Theorem 11-10
Surface Area of a Sphere
The surface area of a sphere is four times the product of π and the square of the radius of the sphere.
S.A. $= 4\pi r^2$ (p. 638)

Theorem 11-11
Volume of a Sphere
The volume of a sphere is four thirds the product of π and the cube of the radius of the sphere.
$V = \frac{4}{3}\pi r^3$ (p. 640)

Theorem 11-12
Areas and Volumes of Similar Solids
If the similarity ratio of two similar solids is $a : b$, then
(1) the ratio of their corresponding areas is $a^2 : b^2$, and
(2) the ratio of their volumes is $a^3 : b^3$. (p. 647)

Chapter 12: Circles

Theorem 12-1
If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency. (p. 662)
- Proof on p. 667, Exercise 36

Theorem 12-2
If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle. (p. 663)
- Proof on p. 667, Exercise 37

Theorem 12-3
The two segments tangent to a circle from a point outside the circle are congruent. (p. 664)
- Proof on p. 667, Exercise 38

Theorem 12-4
Within a circle or in congruent circles
(1) Congruent central angles have congruent chords.
(2) Congruent chords have congruent arcs.
(3) Congruent arcs have congruent central angles. (p. 670)
- Proofs on p. 674, Exercises 23, 24; p. 675, Exercise 35

Theorem 12-5
Within a circle or in congruent circles
(1) Chords equidistant from the center are congruent.
(2) Congruent chords are equidistant from the center. (p. 671)
- Proofs on p. 671; p. 675, Exercise 37

Theorem 12-6
In a circle, a diameter that is perpendicular to a chord bisects the chord and its arcs. (p. 672)
- Proof on p. 674, Exercise 25

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Theorem 12-7
In a circle, a diameter that bisects a chord (that is not a diameter) is perpendicular to the chord. (p. 672)
- Proof on p. 672

Theorem 12-8
In a circle, the perpendicular bisector of a chord contains the center of the circle. (p. 672)
- Proof on p. 678, Exercise 36

Theorem 12-9
Inscribed Angle Theorem
The measure of an inscribed angle is half the measure of its intercepted arc. (p. 679)
- Proofs on p. 679; p. 683, Exercises 40, 41

  Corollary 1
  Two inscribed angles that intercept the same arc are congruent. (p. 680)
  - Proof on p. 684, Exercise 42

  Corollary 2
  An angle inscribed in a semicircle is a right angle. (p. 680)
  - Proof on p. 684, Exercise 43

  Corollary 3
  The opposite angles of a quadrilateral inscribed in a circle are supplementary. (p. 680)
  - Proof on p. 684, Exercise 44

Theorem 12-10
The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc. (p. 680)
- Proof on p. 684, Exercise 45

Theorem 12-11
The measure of an angle formed by two lines that
(1) intersect inside a circle is half the sum of the measures of the intercepted arcs.
(2) intersect outside a circle is half the difference of the measures of the intercepted arcs. (p. 687)
- Proofs on p. 688; p. 692, Exercises 29, 30

Theorem 12-12
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and circle. (p. 689)
- Proofs on p. 689; p. 692, Exercises 31–33

Theorem 12-13
An equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. (p. 695)

Constructions

Construction 1
Congruent Segments
Construct a segment congruent to a given segment. (p. 44)

Construction 2
Congruent Angles
Construct an angle congruent to a given angle. (p. 45)

Construction 3
Perpendicular Bisector
Construct the perpendicular bisector of a segment. (p. 46)

Construction 4
Angle Bisector
Construct the bisector of an angle. (p. 47)

Construction 5
Parallel Through a Point Not on a Line
Construct a line parallel to a given line and through a given point that is not on the line. (p. 181)

Construction 6
Perpendicular Through a Point on a Line
Construct the perpendicular to a given line at a given point on the line. (p. 182)

Construction 7
Perpendicular Through a Point Not on a Line
Construct the perpendicular to a given line through a given point not on the line. (p. 183)